Mathematical modeling of biophysical processes affecting oyster reef development

Rachel Housego
Mentor: Dr. Johanna Rosman
Introduction

• Oysters perform important ecosystem services but populations have declined substantially over the past century.

• Currently there is a push for oyster reef restoration.

• Smothering of oysters by sediment limits the success of some of these reefs.

• Aim of the model is to predict the success of oyster reef restoration projects in environments with different physical conditions.
Erosion
Mortality
Shell
Degradation

Growth
Deposition
Erosion
Mortality
Shell Degradation

Live Oyster
Sediment
Shell
Jordan-Cooley Model

\[ \frac{dO}{dt} = r O f(d)(1 - O / k) - \mu f(d)O - \varepsilon (1 - f(d))O \]

\[ \frac{dB}{dt} = \mu f(d)O + \varepsilon (1 - f(d))O - \gamma B \]

\[ \frac{dS}{dt} = -\beta S + C_{ge} F_c / C_g \]

Oyster Volume (O)
Shell Volume (B)
Sediment Volume (S)

Growth
Mortality
Shell Degradation
Erosion
Deposition
Bifurcation Point

- Bifurcation Point—the point of transition between alternative stable states
- Initial oyster and sediment volumes kept constant at 0.01 m and the initial shell volume was varied.

**Bifurcation Point:** 0.032 m
Revision of the model

• Volume fraction-failed to account for the shell:sediment ratio
• Growth term- negative growth does not make physical sense
• Deposition and erosion- constants were used to model physically variable processes
$h(O) = \frac{O}{\gamma_{O+B}}$

$h(B) = \frac{B}{\gamma_{O+B}}$

$h(S) = \frac{S}{\gamma_S}$

$h = \text{height of layer (m)}$

$\gamma_{O+B} = \text{volume fraction of shell and oyster}$

$\gamma_S = \text{volume fraction of sediment}$

**Bifurcation Point:**

0.032 m → 0.074 m
Growth Term

**Jordan-Cooley Model**

\[ \text{Growth} = r O f(d)(1 - O/k) \]

- Instantaneous Growth Rate
- Carrying Capacity

**Revised Model**

\[ \text{Growth} = A - R \]

- Assimilation
- Respiration

**Variables**

- Food Concentration
- Temperature
- Constant Based on Flow Speed
- Filtration Rate
- Total Particulate Content
- Ambient Food Concentration

**Graphs**

- Jordan-Cooley Growth Rate
- Revised Growth Rate
Revised Growth Term Model Runs

Bifurcation Point

0.074 m → 0.03 m

Volume (m³/m² reef)

Time (year)
Deposition

Jordan-Cooley Model

\[ D = C \frac{\rho_o}{g} \frac{F_0}{e} \]

- Maximum Deposition Rate
- Maximum Sediment Filtration Rate
- Reduction in Deposition with Reef Height

Revised Model

\[ D = c \cdot W_s \]

- Rouse Sediment Concentration Profile
- Settling Velocity
- Grain Size
- Depth of Water
- Frictional Velocity of Water
- Sediment Velocity
- Density of Water
- Viscosity of Water

Graphs:
- Jordan-Cooley Deposition Term
- Revised Deposition Term
Erosion

Jordan-Cooley Model

\[ E = -\beta \cdot S \]

- Sediment Erosion Rate
- Sediment Volume

Revised Model

\[ E = \theta \cdot \Gamma \]

- Destabilizing Term
- Stabilizing Term

- Decay of Shear Stress with Reef Height
- Frictional Velocity
- Shields Parameter
- Critical Shields Parameter
- Sediment Density
- Water Density
- Grain Size

Graphs:

1. **Jordan-Cooley Erosion**
   - \( h_{0+B-h_s} (m) \)
   - \( h_{0+B-h_s} (m) \)
   - Sediment eroded (m^3/m^2 of reef)

2. **Revised Erosion Term**
   - \( h_{0+B-h_s} (m) \)
   - Sediment eroded (m^3/m^2 of reef)
Revised Sedimentation Term Model Runs

Bifurcation Point

0.03 m → 1.67 m

Graphs showing volume changes with time (years) for different categories:
- Oysters
- Shell
- Sediment
- Reef height

Graphs on the left and right side show the progression of volume over time.
Sensitivity Analysis

Bifurcation Point vs. Shell Degradation Rate

Bifurcation Point vs. Ambient Food Concentration

Bifurcation Point vs. Sediment Diameter

Bifurcation Point vs. Velocity of Water
Applicability of the Model

• Final bifurcation point of 1.67 m on a similar scale to successful restored reefs constructed by Peterson et al.

• Identifying the sensitivity of the model to different parameters facilitates understanding of how the model results might deviate from the physical environment.

• Contributes to understanding about how physical processes impact reef restoration success.

• Can be used to identify the relative success of restored reefs under different environmental conditions.
Future Work

- Identifying the cause of the difference in magnitude between the deposition and erosion terms
- Introducing a term that adds the eroded sediment back into the Rouse profile
- Adding a horizontal component to the model
Acknowledgements

• Dr. Johanna Rosman
• Justin Ridge
• Dr. Joel Fodrie
• Dr. Antonio Rodriguez

Questions?